Part I Problems

Problem 1: Use the Euler method and the step size .1 on the IVP $y' = x + y^2, \quad y(0) = 1$, to calculate an approximate value for the solution $y(x)$ when $x = .1, .2, .3$. (Make a table.) Is your answer for $y(.3)$ too high or low?

Part II Problems

Problem 1: [Euler's method] **(a)** Write y for the solution to $y' = 2x$ with $y(0) = 0$. What is $y(1)$? What is the Euler approximation for $y(1)$, using 2 equal steps? 3 equal steps? What about n steps, where n can now be any natural number? (It will be useful to know that $1 + 2 + \cdots + (n-1) = n(n-1)/2$.) As $n \to \infty$, these approximations should converge to $y(1)$. Do they?

(b) In the text and in class it was claimed that for small h, Euler's method for stepsize h has an error which is at most proportional to h. The n-step approximation for $y(1)$ has $h = 1/n$. What is the exact value of the difference between $y(1)$ and the n-step Euler approximation? Does this conform to the prediction?

5/3/25

Part I

1. $y' = x + y^2, \quad y(0) = 1$

$h = 0.1$

n	x_n	y_n	m_n	$m_n h$
0	0	1	1	0.1
1	0.1	1.100	1.2	0.12
2	0.2	1.220	1.42	0.142
3	0.3	1.362		

$y'' = 1 + y'$

$\therefore \; y(0) = 1.000$

$y(0.1) = 1.100 \qquad y'' = 1 + 1 = 2$

$y(0.2) = 1.220 \qquad y'' = 1.1 + 1.2 = 2.3$

$y(0.3) = 1.362 \qquad y'' = 1.22 + 1.42 = 2.64$

$\therefore$ too low

Part II

1. (a) $y' = 2x$, $y(0) = 0$

$$\int \frac{1}{2}\, dy = \int x \, dx$$

$$\frac{1}{2} y = \frac{x^2}{2} + c_1$$

$$y = x^2 + C$$

$$y(0) = 0 \Rightarrow \quad C = 0$$

$$y = x^2$$

$$\therefore y(1) = 1$$

$h = 0.5$

n	x_n	y_n	m_n	$m_n h$
0	0	0	0	0
1	0.5	0	1	0.5
2	1	0.5		

$y(1) = 0.5$

$h = \frac{1}{3}$

n	x_n	y_n	m_n	$m_n h$
0	0	0	0	0
1	$1/3$	0	$2/3$	$2/9$
2	$2/3$	$2/9$	$4/3$	$4/9$
3	1	$2/3$		

$y(1) = \frac{2}{3}$

$h = \frac{1}{n}$

n	x_n	y_n	m_n	$m_n h$
0	0	0	0	0
1	$1/n$	0	$2/n$	$2/n^2$
2	$2/n$	$2/n^2$	$4/n$	$4/n^2$
3	$3/n$	$6/n^2$	$6/n$	$6/n^2$
4	$4/n$	$12/n^2$	$8/n$	$8/n^2$
5	$5/n$	$20/n^2$	$10/n$	$10/n^2$

$n-1 \quad n-1/n \quad \frac{(n-1)(n-2)}{n^2} \quad \frac{2(n-1)}{n} \quad \frac{2(n-1)}{n^2}$

$n \quad 1 \quad \frac{n(n-1)}{n^2}$

$$y(1) = \frac{n(n-1)}{n^2}$$

$$= \frac{n-1}{n}$$

$$= 1 - \frac{1}{n}$$

$$= 1 \quad \text{as} \quad n \to \infty$$

$\therefore$ yes